THE MATHEMATICS TEACHER

A MAGAZINE DEVOTED TO THE
INTERESTS OF TEACHERS OF MATHEMATICS

Volume I | December, 1908 | Number 2

CONTENTS

Editorial 41

The Aims of Studying Plane Geometry and How to Attain Them. By E. P. Sisson 44

Modern Tendencies in the Teaching of Mathematics. By Isaac J. Schwatt . 47

The Aims in Teaching Algebra and How to Attain Them. By C. E. Bikle . 77

Notes and News 79

Published Quarterly by the

ASSOCIATION OF TEACHERS OF MATHEMATICS
FOR THE MIDDLE STATES AND MARYLAND

LANCASTER, PA., SYRACUSE, N. Y.

NEW YORK, N. Y., PHILADELPHIA, PA.

THE MATHEMATICS TEACHER

A MAGAZINE DEVOTED TO THE
INTERESTS OF TEACHERS OF MATHEMATICS

THE MATHEMATICS TEACHER is published quarterly—September, December, March and June—under the auspices of the Association of Teachers of Mathematics for the Middle States and Maryland.

The annual subscription price is $1.00; single copies, 35 cents.

Remittances should be sent by draft on New York, Express Order or Money Order, payable to The Mathematics Teacher.

THE MATHEMATICS TEACHER,
41 NORTH QUEEN STREET, LANCASTER, PA.
OR SYRACUSE, N. Y.

THE MATHEMATICS TEACHER

EDITED BY
W. H. METZLER

ASSOCIATED WITH
EUGENE R. SMITH JONATHAN T. RORER

VOLUME I DECEMBER, 1908 NUMBER 2

EDITORIAL.

The Old and the New.

Modern psychology has done much during the last decade or two in correcting false notions concerning various phases of education. The old idea that the study of some subjects furnished an all-round mental discipline entirely available for the handling of any other subject has been much modified and replaced by the more rational idea of the study of those subjects furnishing somewhat less general and some more special disciplines. It would seem, however, that there has been much misunderstanding and misinterpretation of the results of recent psychological study especially as stated by some writers, if indeed it is not true that some enthusiastic writers have overstated the truth and left the impression that practically nothing gained from the study of one subject is of value in the study of a new subject. If we mistake not the present view—the one arrived at after more sober thought—is that there is a good deal of value derived from the proper study of most subjects which is carried over and is helpful in the study of any other subject and that there are certain elements which are not carried over as available aids in the study of another. Just what elements of the general type and what elements of the particular type the study of each subject furnishes has perhaps not been carefully worked out for many subjects, but it is believed that for all subjects that which is carried over is more important than that which is not. Certainly it is so for mathematics.

In this period of active reform movement in the teaching of mathematics it might be well to remember that not everything that is old should be discarded because it is old and that not everything new is to be adopted because it seems new as a few would lead us to believe. That to which we shall settle down after the agitation is over will be neither all old nor all new, but out of the old and the new together will be evolved that which seems satisfactory to most people and will abide for a period at least. It will also be found that the new elements in the result have been in successful use in the past by individual teachers here and there, whose great aim has been to teach methods and how to apply them.

The Perry Movement.

A good deal has been written and said in this country concerning this movement, but Professor Perry complains that in England his ideas are little understood and we believe they are no better if not less understood here than there, due it may be to the difference in conditions. The latest pronouncement with reference to his ideas is perhaps that made by him at the joint meeting (held November 28 in London) of the MATHEMATICAL ASSOCIATION and the FEDERATED ASSOCIATIONS OF LONDON NON-PRIMARY TEACHERS. A very complete account of this meeting is given in the January number of the *Mathematical Gazette* from which the quotations which follow are taken. Professor Perry says: "It astonishes me to see how little comprehension there seems to be of the proposals made by the British Association Committees. We recommended experimental geometry with common-sense reasoning, and everybody seems to think that we asked for a babyish use of rulers and compasses following a series of propositions. We recommended some work with graphs on squared paper and some teachers do nothing but graphs, and there are dozens of school books to help on the craze. It reminds me of a friend who does so many things with cement that I almost think if he wanted a new umbrella he would make it of cement. Some teachers think that squared paper was invented merely to illustrate the solution of certain simultaneous or quadratic equations. I do not say that some

teachers are carrying our reforms too far; I say that they do not at all comprehend the nature of the reforms suggested by the British Association and other committees."

Again speaking of his ideas he says: "I suppose we must consider the cases of: (1) The training of all children, the citizens of the future being supposed to be taught science for various good reasons: one, that he may take an interest in those things which are more important in transforming the world than all others: another, that he may learn scientific method, which will cause all his actions to be more efficient. Every man who has done important things in history was distinguished by his having scientific method, not perhaps gained in our modern way. We want the average man to have it now. (2) The training of the boy or man who intends to enter a profession of applied science. (3) The training of average men who intend to teach mathematics and science. (4) The training of a few mathematicians and scientists who may possibly become eminent, and may or may not become teachers. *I do not think that the early training of any of this class should differ from that of the others.*"* He feels that a boy should not wait until he can rigorously demonstrate a theorem before using it and says: "It is to be remembered that in the days of great scientific development, the days of Leibnitz and the Bernoullis, the days of Lagrange, the days of Fourier, powerful methods of analysis were used which were not proved to be legitimate until long years afterwards. * * * A student ought to use logarithms long before he approaches the proof of the exponential theorem. He ought to use Taylor's theorem long before he studies what the mathematicians might call a rigorous proof of it." Again he says: "To keep ourselves right, to prevent fashion and habit or bad example from leading us astray, we ought to stick to one great principle—*our object is not to convey information, or a thousand labor-saving rules, our object is to teach scientific method.*"* "Accuracy in algebra, accuracy in measuring, each of these is as essential as the other to the success of our reforms." It should also be remembered that Professor Perry does not expect the reforms to be brought about under present conditions, for he says before it can be brought about: "Salaries must be doubled and forms halved in their number of boys."

* Italicised by editor.

THE AIMS OF STUDYING PLANE GEOMETRY AND HOW TO ATTAIN THEM.*

By E. P. Sisson.

"There is no royal road to geometry," says Euclid. Plato had written over the porch of his famous school, "Let no one who is unacquainted with geometry enter here." An old poet has written, "Without enthusiasm, no mathematics."

Geometry is a human book—not divine; therefore a very imperfect book. Geometry is the product of the human mind, and not of the hand; therefore the subject concerns the intellect, and is not mechanical.

The aims of studying geometry in our schools are many; but in my opinion the most important purpose is the training of the reasoning faculty, which gives the student the power of clear and accurate thinking, and the power to express his thought in concise and elegant language. A story is related by Stobæus which runs thus: A youth who had begun to read geometry with Euclid, when he had learned the first proposition, inquired, "What do I get by learning these things?" Euclid at once called his slave and said, "Give him three pence, since he must make gain out of what he learns."

Another purpose of studying plane geometry is to obtain the knowledge by which the student can measure lines, angles and surfaces. The student of plane geometry lays the foundation necessary for the study of solid geometry, and the study has also a practical application in the study of the sciences, especially chemistry and physics. The study of geometry gives to the mind what the strope does to the razor—a keen, cutting edge.

In my opinion, fully three fourths of the value of the study of geometry is disciplinary. I well know that this is not in harmony with the modern thought and method. We meet those teachers occasionally who say that the study of geometry is becoming stale and uninteresting, and therefore we must have something new, in method at least. I profoundly pity such teachers. Our experience and study leads us very emphatically

* Read at the meeting of the Syracuse Section, December 29, 1908.

to the conclusion that science precedes all art, and that theory goes before practice and the general formula includes all particulars. The mind is strengthened far more by the consideration of general propositions than by the consideration of isolated ones. The mind must ever direct the hand.

How shall we as teachers present the subject of plane geometry so our pupils will get the most out of the study that will be profitable to them in after years? Any method must be very flexible, in order that it can be adapted to each individual teacher. I venture to suggest a method something like the following. First, decide on the text-book to be used. Second, commit the definitions, absolutely, of the fundamental ideas in that text, such as point, line, extension, direction, geometrical figure, angle (both as a figure and as a magnitude). Be sure that the class understand clearly and accurately the language used and can illustrate each definition. Third, commit the corollaries from the definitions. Fourth, the axioms and postulates.

I wonder how many teachers ever asked their pupils to state in good English what they understand by mathematical reasoning. I have never read in any geometry a definition of mathematical reasoning. In my mind, the definition is something like this: Mathematical reasoning consists of a series of mental steps by which the mind passes from the hypothesis of a proposition to a conclusion, and each step is made up of two distinct parts; first, the statement of a fact that leads the mind toward the conclusion, and, second, the authority for such statement, which must always precede the proposition under consideration, and the authority must be a definition, corollary, axiom, postulate, or a proposition previously demonstrated. "Is evident" is never an authority for any step in a mathematical demonstration. Olney says, "The mathematical reasoning by which the truth or falsity of a proposition is made to appear is called a mathematical demonstration;" but, mark you, this does not define mathematical reasoning.

We find no two authors agreeing in the definitions, corollaries deduced from the definitions, axioms, postulates, and order of propositions; therefore the demonstrations will differ in the method, order, and number of steps required, in accordance with the text-book used. The following will illustrate how

authors differ in the first proposition. The first proposition in Wells is: If two straight lines intersect, the vertical angles are equal. In Beman and Smith: All right angles are equal. In Olney: At any point in a straight line one perpendicular can be erected to the line, and only one which shall lie on the same side of the line. Robbins: A right angle is equal to half a straight angle. Lyman: If two triangles have two sides and the included angle of one respectively equal, the triangles are congruent. Euclid: Upon a given segment line construct an equilateral triangle. Stewart: To make two straight lines coincide in direction. Gillette: All the radii of a circle are equal. Newcomb: A straight line can be bisected in only a single point. Pettee: If one straight line meets another, the adjacent angles are supplementary. This is sufficient to illustrate my point. We find almost as great a difference in the definitions, postulates, corollaries, axioms, as given by different authors.

After having decided upon the text-book to be used, have the class work from four to six weeks without any text-book except as they use it for the definitions, axioms, postulates, and corollaries. During this time, dictate from ten to fifteen fundamental propositions that can be easily demonstrated by the use as authorities of the definitions, corollaries, axioms, and postulates already given; and have the class go over each demonstration carefully and thoroughly in order to show to the student the "nature of a geometric proof, and to lead him by easy steps to appreciate the logic of geometry."

After this, follow the text-book closely, and have the class see that the author is consistent in his development of the subject in accordance with his definitions, axioms, postulates, and corollaries, and order of propositions as dictated and given by the author used.

The recitation should be the opportunity for the teacher to direct the work of the student, but, under no conditions, to *do* the work *for* the student. Too often the recitation period is spent in talking about the subject, which is time worse than wasted. In general, no student in geometry should be allowed the privilege of the class without having first studied hard, from one to two hours if necessary, upon the lesson previously assigned. Give the student just as little help as possible, and eliminate all material that is not absolutely essential in each

demonstration. This will not make the subject of geometry interesting or a pleasure to every student. Some will dislike it for a time, and a few will fail; but we believe in the end more will be helped than discouraged by this method, and a much higher grade of scholarship will be attained.

COLGATE ACADEMY,
HAMILTON, N. Y.

MODERN TENDENCIES IN THE TEACHING OF MATHEMATICS.*

BY ISAAC J. SCHWATT.

Until the middle of the last century, mathematics had been developed only from the top, so to speak; but during the last decades, the efforts of some of the ablest mathematicians have been directed towards obtaining clearer conceptions of the foundations of mathematics.

Cantor, Weierstrass, Dini, Dedekind and many others have studied the fundamental conceptions of number and space, and their work has resulted in more accurate ideas about these conceptions.

With all the advance in the knowledge of mathematics, with our more thorough conceptions of the foundations of the science, and with the resulting tendencies to change the methods of presenting its various branches, the effect on the student's knowledge of the subject is far from encouraging.

During the last few years, quite a little unfavorable criticism has been made on the efficiency of our schools. Many of these criticisms refer to the results we now obtain as compared with those obtained before. We do not concern ourselves with the criticisms made by skeptics; they have been, and they always will be with us. Nor ought we to take seriously the statement by George Bernard Shaw: "Those who can, do; and those who cannot, teach." But there have been criticisms made by members of our own craft. Professor Charles W. Larned, of the United States Military Academy, in an article entitled, "The Inefficiency of our Public Schools," *North American Review*, September, 1908, tells us that out of 314 candidates who submitted

*Read at the December meeting of the New York Section.

in June, 1908, to the examination for entering West Point, 154 failed in algebra, and 237 in geometry. The examination questions were not difficult. They were free from what are often termed 'catch questions,' and have impressed me as being eminently fair. I understand that during the examination the candidates are very closely watched, and that they have to depend entirely on their own resources.

The results of the examinations in other subjects are correspondingly lamentable. This is the more surprising since, as a rule, only the ablest candidates are chosen, and many win their appointment by competitive examination. The candidates represent a class of earnest men, 206 out of the 314 having either wholly or in part earned their own living while at school and while preparing themselves for the Academy.

In 1905 the Association of Mathematical Teachers in New England addressed letters to teachers of physics and chemistry throughout the schools and colleges of New England, inquiring as to their students' ability to perform arithmetical and algebraical operations.* One of the questions of this inquiry reads: "If you have taught for some time, please state whether you notice any difference in the pupil's preparation in mathematics now and formerly."

Some of the answers would indicate that the pupils have deteriorated under the modern methods of teaching, or whatever the cause might be.

Professor Barus, of the Department of Physics, Brown University, writes: "There is a change for the worse. Coming from the high school of Cincinnati to Columbia College thirty-one years ago, we were decidedly better prepared in mathematical subjects than the men I teach now within forty miles from the 'Hub of the Universe.'"

Professor Bartlett, of the Department of Chemistry, Dartmouth College, reports: "I refuse to teach arithmetic. . . . Most of the students are helpless. Incorrect calculations are so common that the first attention is always directed to the calculations in case of erroneous reports. . . . Problems involving a little algebra usually throw down the whole division . . . . I have been teaching here for twenty-six years and had four

* Third report of the Association of Mathematical Teachers in New England, 1906.

years experience in secondary schools previously. I think students are much less competent in arithmetic now than formerly."

As a member of the teaching profession, I feel keenly the humiliation and the sting of the criticisms on the results of our efforts. If the president of one of our leading universities can give expression to the following: "We all know that the children of the last two decades in our schools have not been educated. With all of our training, we have trained nobody. With all of our instructing, we have instructed nobody"—there must be something decidedly wrong with the teaching of the subjects of the curriculum and among them with the teaching of mathematics, which is acknowledged to be one of its strongest educational mediums. I am sometimes surprised at the apparent indifference and complacency with which we teachers in general seem to take these criticisms. If the criticism on the efficiency of our schools is justified, it is our duty by single and united effort to remove the cause of it. If education is good at all, it must be effective, and its results must be commensurate with the high importance we attach to it, and the large amount of money we expend on it.

Whether the fault lies with us teachers, or with those who are charged with the administration of our schools, or with the object of our efforts, the pupil, upon us devolves the duty to study the conditions earnestly, carefully and persistently, and to bring about such changes as will improve the existing conditions.

It is pathetic to think that our efforts have not been crowned with better success. On all other questions affecting the welfare of the community and the state, every one of its members has his say. But the average man or woman pays very little attention to the work of the teacher. We teachers have it all our own way. We can make up our curriculum and choose our methods of teaching without interference from the public. The public judges us by the results of our labors only. It is therefore the more our duty to do our best and to see that our efforts shall be most effective.

In a paper entitled "Our Duty as Teachers," I have considered one of the causes for the apparent failure of our efforts. This cause is the tendency of the student to conceal his ignorance from his teacher, instead of frankly trying to obtain all possible help in his difficulties.

There was a time when the peculiarities of the teacher were of such a nature as often to enable a pupil, who was not true to his own interests, to pass a subject on insufficient knowledge. The pupil took advantage of these peculiarities of the teacher who was easily led to have a higher opinion of the pupil's diligence and knowledge than he deserved. But the day of such teachers has passed. They survive only in the comic papers. The teacher of to-day ought to be well able to cope with the pupil who hides his ignorance intentionally or otherwise.

In this connection I am reminded of a little story which is quite characteristic of the old-time professor. It is told of a professor in one of our largest universities, that while walking on the campus and meditating upon a problem, he met a student. Linking arms with the student the professor proceeded to explain the problem to him and to demonstrate its solution. When he was at the end of the demonstration, he said to the student, "Don't you see that the result is (let us say) a?" The student, who had not paid much attention and was, in fact, unable to follow the demonstration replied with surprise, "Is it?" The professor, believing that the student while following the demonstration had obtained a different result, went over the demonstration, and found that his first result was incorrect. The apparent knowledge and ability of the student so impressed the professor that from that time on, the student ranked in the mind of the professor as one of the ablest mathematicians in the university, always receiving the highest marks in the subject.

The failure of the pupil to derive from his studies the mental development, for which they are intended, renders futile, at least in part, the aims and the purposes of education.

What then is the purpose of sending children to school? What, in short, is the purpose of education? On the answer to this question depend the subjects to be taught in the school, the method of instruction, and the entire attitude of the teacher towards the student.

In answering this question, we shall exclude from our consideration the professional schools, the industrial and trade schools, etc., and shall consider the elementary and such secondary schools and colleges as give what we term liberal courses only.

Now, as I understand it, the purpose and aim of education

should be the physical, moral and intellectual development of the young, whose fitness shall determine what the world is to be in time to come.

It is true beyond discussion, that in order to do right, to be useful to ourselves and to our fellowmen, we must first and above all be healthy and strong. Our bodies must have the power of resistance to enable us to discharge the manifold duties incumbent upon us.

The school should make provision for the physical development of the child. The school ought to teach the laws of hygiene and the rules of right living. The child must be taught that the proper care of the body is as important as the care of the mind, that one of the means for keeping the body in perfect health is the use of proper and well prepared foods, of pure water, pure air, etc.

The second function of the school ought to be the formation of character, the inculcation of moral habits, of habits of right acting, and of all that is true and noble and just.

Most schools seem to consider the intellectual development of the youth as their most important, if not their only function; but as the motto of the University of Pennsylvania, "Literæ sine-moribus vanae" (learning without morals is vain), emphasizes the fact that, one of the most important functions of the school ought to be the development of character, the inculcation of habits of sincerity and frankness, of diligence and perseverance, of honesty of purpose and application to duty, of reverence and kindness and consideration of others.

A person possessed of these qualities and only moderate intelligence is a more desirable member of society and is bound to succeed better in life, in the real sense of the word, than a person who is lacking in some of them, no matter how great his intelligence or how strong his mental powers might be.

We must take into consideration that there are many parents who have not the time to supervise the moral education of their children. There are persons who have assumed the responsibilities of parenthood without being well fitted for this great task. On that account, very often they do not enjoy the reverence of their children. The latter, as a rule, show more respect for their teacher than for their parents. With such children especially lies the great power of the teacher, and his opportuni-

ties and possibilities to develop and influence character. Moreover since the child spends the greater part of the day in school and in the preparation of his lessons, there is not much time left for even the intelligent parents, with high moral ideas, to conduct the moral education of their children. The duty of moulding the character of the child therefore devolves, in part at least, upon the school and the teacher.

It is perhaps a fortunate circumstance that in most schools no formal provision is made for the moral education of the child and no dogmatic teaching of morals is included in the curriculum. If without preaching to the child, we use every opportunity to draw a lesson in morals and character building, the purpose will be much better accomplished.

We now come to the third function of the school—the intellectual development of the young. We believe in the goodness of human nature, and in the instinctive desire of every individual to do right, if he can only think right. I am in full accord with Madame de Stael, that all bad actions are only the result of thoughtlessness. Mental development, power of mind, the ability to think right, are the first requisites for character. If we could only think right in all matters pertaining to ourselves and to our relations with others, we would also act right.

There was never a time when people were so easily convinced by arguments without submitting them to the careful scrutiny of their reason and understanding. People seem now more willing than ever to have their thinking done for them. There was never a time when the power of the self-constituted leader who takes it upon himself to do the thinking for the people, was so great as it is to-day, when all kinds of sects of religion and all kinds of social theories were so prevalent; when the influence of the "walking delegate," whose office is to hand out ready-made ideas to his constituents, helpful or baneful, was as strong as it is to-day.

Just as we must submit all our actions to the test of the highest standards of morals, so we must submit everything we learn, read, or hear to the test of our understanding. We very often think that we understand an idea, but when we come to express it, or carry it into execution, we find that the idea is not entirely clear to us.

One of the most pernicious habits of the mind, against which

we must constantly and most vigilantly guard our students, is that of superficiality; of accepting an idea before submitting it to the test of his understanding; of being satisfied to take up another idea before the first one is entirely clear and assimilated.

This habit of superficiality is only a prelude to a slip-shod way of doing things, to be satisfied when a task has been done so that the result will look right, yet not stand close scrutiny or careful examination. To do every task right and to the best of one's ability is an important and essential quality in every occupation in life. The time to acquire this all-important habit is in childhood, and the place to begin such a preparation is in the classroom, whatever the subject taught.

Our number system, which is so well adapted to the performing of arithmetical operations in a more or less mechanical way, and in this lies the claim for the superiority of our decimal system over all other systems, has contributed a great deal towards the pernicious tendency of doing things mechanically, without much exercise of the power of mind.

The pupil carries this tendency to his work in all the other mathematical subjects. Since the young find concentration of the mind the hardest task, they easily give way and do things mechanically wherever possible.

While the method of the ancients, like the Egyptians and the Romans, for carrying out arithmetical calculations were rather cumbersome, one cannot help thinking that these methods were more conducive to the acquiring of mental power, than the refined methods we apply in our calculations to-day.

A child who recites a poem without understanding its contents receives very little benefit from memorizing it. The pernicious habit which the child acquires of saying things without understanding them, is a danger to its moral and intellectual development. We must, above all, educate the children to reason and to think. While thinking is the hardest work, we can so cultivate this all-important habit of the mind in the child that it will enjoy reasoning, and find thinking an agreeable task. No person can discharge his duties of citizenship unless he is able to reason correctly, and to think right.

It is the duty of the teacher to see that the student has a clear understanding of every idea presented to him, and that he acquires the habit of submitting every idea to the test of his

reasoning before accepting it and calling it his own. Every idea introduced must be of such a nature that the student can gain a clear understanding of it, his age and mental development being taken into consideration.

It is the duty of the school and the college to pay special attention to those students who are less gifted, or who are lacking in the sense of duty, and have not the strength of character to apply themselves to their studies. If schools and colleges drop young men from their rolls because they do not show aptitude and inclination for work, or because they are unruly, such youths without our help and encouragement may become a burden to themselves and to those around them. If they do not do their duty in their work in life, there are other men and women eager to take up their positions, and with little sentiment they are simply dismissed.

It is the special duty of the school to improve such youths, to strengthen their character, to build them up mentally, and if possible, make useful men and women of them. In the long run, we shall pay the penalty if we neglect to help those who need our help most.

The student who leaves school or college because he is not successful in his studies often does well when he engages in manual work or in an occupation which requires little mental effort and little continuous exercise of his power of thinking. If his failure at school is due to his lack of ability, it might be better for him to continue his studies, and by proper training and direction acquire the power of mind, without which his usefulness as a citizen must necessarily be impaired.

All the functions of the school are so interrelated that if one is neglected the others are bound to suffer. To do right, we must think right, and to think right we must be healthy in body and mind. Mens sana in corpora sano. To be healthy we must have an easy conscience, we must be able to decide and choose what is good for our mind and body.

The satisfaction resulting from having done work right is invigorating and is conducive to true happiness. Some of us perhaps remember the worry we experienced when we were not sufficiently prepared in our lessons, and the anxiety we have felt lest we should be called upon to recite. Those children who are diligent and have all other good qualities, and are there-

fore free from anxiety and fear of punishment, are as a rule the healthiest and happiest. What is true of adults is true of them. If we are successful in our efforts, we are happy and as a rule healthy, however hard we may work. It is worry and not work that impairs our health. If we all think right, we shall act right, and we shall be less subject to worry, and more endured to work.

To acquire knowledge, the student must possess certain traits of character. He must have the qualities of diligence, sincerity and perseverance. If he is lacking in these, it is the duty of the school to develop them, not only because they are all-important for every occupation in life, but because without them his mental development is not possible. Any failure to awaken and develop these qualities in the pupil and the consequent failure to derive the mental benefits from his studies, are bound to cripple his character for life.

The character of the child which is still passive, with no expressed tendency for good or evil, can be formed in such a manner as to acquire habits which will influence his life for good. If he acquires while at school the habits of diligence, of being punctual, of being sincere, of being perseverant in his work, he is most likely to carry these habits to his work in life. If he is, however, negligent in performing his duties while at school, he is likely to shirk his duties when he is a man. If he pretends to know what he really does not understand, he will acquire the habit of insincerity. If he is quieting his conscience and making himself believe that he understands an idea without giving his understanding a searching and scrutinizing test, this will tend to create in him the disastrous habit of superficialty. If he is inclined to be irregular in attendance while at school, he is likely to show this same tendency later as a man. What an injustice to the child who will have to pay the penalty for any failure in his character due to the neglect of the teacher!

We must teach our students that diligence, perseverance and sincerity are as desirable qualities in the young as they are in the grown-up person; that there is only one step between school life and what we call practical life; that since the habits acquired in youth are most lasting, the pupil must acquire the habits of right action, and practice them in all his activities while at school.

The methods of education ought to be the same whether the child is to become a laborer, a mechanic, or a professional man.

The standard of morals and the ability to think right and to act right ought to be the same whatever may be the station or the occupation of a person in life. Just as there is one legal code, there is only one moral code, and we all have to conform to its teachings. Whatever advantages outside of the school the child of the well-to-do may enjoy over the child of the poorer classes, each must have in school the same advantages for his physical, moral and mental development, if he is to be able to make use in the same measure of the opportunities, which our country offers to all its citizens.

The stability of this republic depends on the ability of the individual to thing and to act right and on his proper conception of happiness, and in what ways true happiness and enjoyment in life can be attained.

The unsatisfactory results of our efforts have caused teachers of mathematics, and especially associations like this to look for remedies. This dissatisfaction has made itself felt in certain tendencies in the teaching of our subject.

In the following we shall discuss some of the aspects of these tendencies and make a few suggestions, in the hope that those who do not agree with the views herein presented, will yet consider them of sufficient weight to continue their discussion, and that some conclusions may be reached which each and every one of us will endeavor to apply for the betterment of the teaching of our subject.

There is a demand for better text-books, more thorough and more lucid, for syllabi to serve as guides in determining what to choose in the teaching of a subject and what to omit. I say it with a great deal of hesitancy that in most cases the preparation of a syllabus is a misdirected effort, and that the use of a syllabus may have an ill effect on the true purposes of teaching. With the increase in the number and in the variety of text-books for all grades in the same subject, a syllabus can be dispensed with.

Moreover I feel that, as a rule, the purpose of a syllabus is to serve as a guide as to what topics some of the colleges expect candidates for admission to be most efficient in. But since the great majority of secondary school pupils do not enter college, a syllabus has no place in such schools.

There is a tendency to represent the subjects of the curriculum, including mathematics, in such a manner as to incite the interest of the learner. But if the young are brought up with the idea that everything they do must incite their eagerness or curiosity, there is no wonder that those who fail to secure work which is in accord with their tastes are augmenting the ranks of those whose theories may mean ruin to civilization and to the progress of mankind.

Those are blessed who have found their work. The large majority of men and women, however, must find the inspiration for their work, and for doing it right, in the consciousness that they are doing something to help the common weal; that it gives them the means to provide for their own needs and for the needs of those who are dependent on their support, that if they do their duty to the best of their ability, they may in time find work which is more congenial to them.

The child and the youth ought to be taught to do their duty. The teacher should insist upon such a course on the part of the student with firmness and strictness.

The methods by which children are made to acquire the habit of doing their work and doing it promptly and well, will differ with the individual child. Some children will give heed to a word, others will be influenced by persuasive talk, again others will do right, only after more or less severe punishment.

What an injustice to the pupil, and what harm is done to his entire future, if the teacher does not insist that he be accurate, punctual, and diligent, and that he devote himself with all his energy to his work. The pupil when he leaves school to-day and finds employment to-morrow will be judged by his conduct and by his diligence. If he fails to comply with the requirements that govern his employment, he may lose it, which may mean humiliation and disappointment, and may result in sad consequences for him, and for those nearest to him.

The function of the school is to prepare the child and the youth for life, as life is, or as it ought to be. The pupil and the student of to-day are the employer and employee of to-morrow, and what they will be, the world will be. The pupil ought therefore to be taught that all of his actions, while at work or at play, must conform to the highest moral standards.

He must understand that to go to school is an occupation, that it is as important for him to be successful in this occupation, as for his father in that which he pursues, and he will be the better fitted for his work in life the better he applies himself to all phases of his school life.

The best preparation for life is to make the young do that which requires effort—effort of thought, effort to overcome any possible disinclination to work; to derive pleasure from doing one's very best and from the satisfaction of overcoming difficulties.

Life is a continuous rendering of service. Service in order to earn a livelihood; service in bringing up our children and in caring for those around us; service in taking part in the higher purposes of life—moral, social and intellectual.

Wherever we turn, there are conditions to improve and work to be done, which call for service, hard and unremitting service, and very often with only the reward that comes from the consciousness of having done something worth while.

There is a tendency, even in the secondary schools, to present the various subjects of mathemathics with a view to their possible application in life, believing that education in order to be effective, ought to be practical.

It is, however, a matter of experience that we soon forget whatever we learn at school, and it is true beyond question that in life, we need more than anything else, health, character and power of mind, for on these our success will largely depend. We can get along with very little mathematics, very little geography and science, very little of the other subjects in the curriculum, if we have only character and the ability to think right.

Even professional schools can give their students only such knowledge as forms the foundation of their special science and of what they will meet in the practice of their professions. The school ought to imbue its students with the scientific spirit and with the thirst for knowledge. When they enter their profession they will have continually to study and to learn. Any change in the hypothesis of their science, every new discovery and invention in it they will have to apply, if they do their full duty.

The great majority of men and women will have little use in life for mathematics, beyond the operations with integers and simple fractions. It is greatly to be regretted that even in the engineering profession, mathematics is being used less and less, in the actual solving of practical problems. Some of the larger manufacturing establishments, such as the Cambria Steel Company and the Carnegie Steel Company are publishing handbooks for engineers which contain formulæ and all kinds of data which an engineer is likely to need in his work. These books have created a class of engineers called "handbook engineers" who use the formulæ and the tabulated data very often without a knowledge and understanding of the methods by which they have been obtained. The use of such books tends to lessen the mental activity of those who use them.

A large engineering company has lately provided its engineer with slide rules, expecting them to be used in making computations, and thus to save time.

But just as it is impossible to prepare the student of engineering in all problems which he may meet in practice, so it is impossible to include in a handbook all the data he may need in his profession. In some cases he will have to fall back on his mathematical knowledge and the more thorough it is the better fitted he will be for the work requiring mathematical theories, the results of which are not to be found in a handbook. The world is a vast store of undiscovered energies and opportunities. We must only sharpen our minds to discover them, and acquire the knowledge and ability to exploit them for the benefit and comfort of mankind.

The greater usefulness of the college-trained engineer compared with the engineer who has learned his profession in the factory and in the field, the so-called practical engineer, lies in the higher development of power of mind in the former and in his ability to use such methods and results as his knowledge of mathematics and mechanics affords.

In manufacturing the aim is to produce an article in the shortest possible time, with the least expenditure of material, but with due regard to quality and efficiency. Now the practical experience and his natural instinct will often tell the engineer in what way a saving of material, etc., can be effected, but in

many cases, such a knowledge can be obtained by mathematical theory only.

Take for instance, the manufacture of reservoirs, of basins, or simply of tin cans. We can construct tin cans of different shapes, and therefore from different amounts of material, but all of one and the same content, a quart, say. Now, unless there is a reason that a can should be of a certain shape (as a can used for baking powder, or for spices, etc.) it ought to be constructed, for a given volume, with the least amount of material, *i. e.*, with the view to economy. Mathematical calculations lead to the result that to construct a can having a top and bottom, the least amount of material will be required if the diameter of the base is equal to the height of the can, and the larger the diameter of the base of a cylinder with neither a top nor a bottom, the less material will be necessary to construct it.

As a result of the tendency to make the work in arithmetic more interesting and practical, the child often learns how to use a commutation ticket or a mileage book, and the names of the parts of a cattle in the language of the butcher, but does not get the mental training which can be obtained from a proper study of the subject, nor acquire the ability to perform with accuracy some of its simplest operations. Such a course is as much to be condemned as the tendency of some people to provide themselves with luxuries, without being able to satisfy the more substantial necessities for their body and mind.

There are so many faculties and abilities to be developed in the young, that the school cannot accomplish even the smallest part of them. It would certainly be most desirable to teach the children to model, and to paint, and to play a musical instrument, and to sing.

It would be highly desirable for the school to discover any latent talents and abilities of the pupil. Some persons have not been aware of possessing high talents, which would have remained latent if they had not been discovered by accident. But until the school can fulfill well the great and difficult tasks of developing the body, the character, and the mind of the young, no other functions should be included in its work.

The schools cannot teach all that is worth knowing, all that we ought to know in order to understand social and economic conditions and changes, the workings of nature and to enjoy

its beauties. But the school ought to awaken the desire for knowledge. It should train the young in power of observation, in power of discrimination, power of concentration, and in all those abilities and qualities of the mind which are developed by the study of the different branches of the school curriculum, and in no small measure by the study of mathematics.

There is a tendency to correlate the different branches of elementary mathematics, especially algebra, with some of the branches of the natural sciences, particularly with physics, and mechanics. There are a number of causes which have led to this tendency, one of them, I feel, has as its base the dissatisfaction of teachers of physics and engineering with the average student's ability to perform with facility and accuracy algebraical and arithmetical operations, and to solve simple equations for the different letters which occur in them. Also, the equations in most of the books on algebra have integral coefficients, and give for the value of the unknown, integers or simple fractions. But many of the problems in physics and engineering lead to equations with decimal coefficients, and whose solutions are sometimes complicated fractions which must be expressed in decimals. I claim that the student who is well trained in performing arithmetical and algebraical operations and who has a clear conception of the principles underlying these operations, ought to be able to handle algebraical expressions, and to solve the equations to which many of the problems in physics or engineering may lead, whatever the coefficients of the equations might be.

There are three kinds of problems from physics and mechanics that might be used to illustrate mathematical operations. To understand the conditions of one kind of these problems, a thorough knowledge of the principles of the physical and mechanical phenomena involved is necessary, but to effect their solution only the simplest arithmetical operations need be used. The solution of such problems belongs in a course of physics. The age and mental development of the pupil when he studies arithmetic will as a rule not permit of the thorough understanding of the principles of physics and mechanics necessary to translate the conditions of these problems into the language of arithmetic.

There are problems from physics and mechanics in the solution of which the methods of algebra may be used to advantage,

and in which the translation of the given conditions leads to very simple algebraical expressions. The solution of this class of problems also belongs more properly in a course in physics. By introducing into such problems conditions which are artificial, the translation of the conditions and the resulting algebraical expressions may be increased to any degree of difficulty.

The great majority of problems from physics and engineering, some even of the simplest kind, need for their solution a knowledge of calculus, and many of them, a knowledge of the more advanced branches of mathematics, like differential equations, definite integrals, etc.

To solve these problems, perfect facility in handling algebraical and trigonometrical expressions, as well as a thorough knowledge of the conceptions and operations of calculus is necessary.

I feel that the cause for the correlation between the mathematical subjects and those of some of the natural and applied sciences lies also in the desire of the teacher of mathematics to satisfy the tendency to make the study of the subject practical and interesting to the student. But most of the problems from physics and mechanics which I have seen or have been able to compose, and the solution of which can be effected by algebraical operations, are of little practical use. There is not much more practical information in figuring out how long it takes a boat to cross a river under certain conditions of wind and current, than there is in figuring out how old Ann is. To calculate how long it would take a boat to actually cross a river, it would be necessary first to determine the velocity of the wind, the velocity of the current, the weight of the boat and so on. To obtain some of these data, a knowledge of meteorology, hydrodynamics, etc., and skill in handling delicate apparatus are necessary.

The entire question as it appears to me is whether the formal study of mathematics, especially of algebra, has its place in the curriculum, and whether the formal study of this subject helps to develop qualities of mind which are desirable.

Just as it requires different ingredients to make up the diet for the body to keep all the organisms healthy and strong, and in a harmonious state of development, so the mind needs for its development, a curriculum embracing different subjects.

The study of formal algebra, without any regard to its appli-

cation, teaches continuous concentration of attention, unconscious following of a number of rules in more or less rapid succession, and is an excellent discipline for acquiring habits of precision, accuracy and power of mind.

Facility in handling algebraical expressions is a necessary preparation for all other branches of mathematics. Without a thorough knowledge of the principles of algebra and perfect facility in performing arithmetical and algebraical operations, the student will not be able to gain a knowledge of the principles of trigonometry, solid geometry, calculus and other branches of mathematics, and to solve the problems of these subjects. The student will need all the time which is ordinarily given in the secondary schools to the study of algebra, to acquire perfect facility in this subject, and not until he has attained such facility, should other matter be introduced.

No student can hope to acquire a thorough mastery of a branch of mathematics unless he is able to perform with little effort such operations from other branches as may be needed in this one.

When he takes up the study of algebra, the student must be able to perform with ease such arithmetical operations as he may need in algebraic work. In the study of trigonometry he must have perfect facility in such algebraical and geometrical work as may be required in the solution of the trigonometrical examples. Again, if he is to successfully acquire a thorough knowledge of the principles and operations of calculus, he must have perfect mastery of such arithmetical, algebraical, geometrical and trigonometrical work as he may need in the study of calculus.

Just as when we have acquired facility in spelling and writing, we need pay little attention to these, but may concentrate our efforts on the formulation of our ideas.

Unless the results of a problem are accurate, they are of little use. To carry a mathematical operation through all of its stages to an accurate result, requires a thorough knowledge of the subject and a facility which can only be acquired by constant practice and by hard work. It is a great injustice to the student who chooses a profession like engineering, if the time which he needs to acquire facility in performing algebraical

operations, is used in the solutions of problems which are to arouse his interest, or which appear to him to be practical.

The educational value and the mental development which is derived from the study of physics is in a sense different from the educational benefit to be derived from the study of mathematics. There can be no objection to the use of occasional illustrations from physics or mechanics, or from any of the sciences, to show the student the power of mathematical study, outside of its value as an educational medium, but the student must have a thorough knowledge of the principles and laws of the sciences from which the illustrations are drawn in order to translate intelligently the conditions of such problems in the language of mathematics.

In the teaching of mathematics, we must be very careful not to give way to fads or to apply new methods and tendencies, unless we are sure that they will lead to more effective results.

We must not experiment on the child that has but a single opportunity to gain or to lose. In the interest of the children who are to make the world what it will be during the next generation, and in a way, during all generations to come; in the interest of the larger purposes of true education, I wish to sound a warning that we exercise, as far as the teaching of mathematics is concerned, the greatest care and caution before applying a "modern tendency" to the minds of the young. For years we have been following in some of the elementary branches of mathematics, the German methods of teaching. There is now a decided tendency to follow Professor John Perry, of the Royal College of Sciences of London, who terms the German method the "intellectual Strassburg goose-stuffing," and in another place, the "intellect by order of the authorities." He calls the English system "antiquated," and has a method all his own.*

This country has proven the most fertile soil for all kinds of experiments in the teaching of mathematics. We have been quick to adopt methods, especially if advocated by foreign pedagogues.

Our course in this respect has been similar to that of a person who attaches a higher appreciation to a thing, even if of inferior make and quality, simply because the label states that it is made in a foreign country.

* *School Mathematics*, March, 1904, p. 195.

The Grube system, long after it has been discarded in the land where it originated, is still in use in many schools here. While the method of teaching and the selection of text-books are in some foreign countries in part regulated by the Government, and therefore more or less uniform, it is left here, as a rule, to every city, and in some places to every school, to choose its text-books and the method of presenting the subject.

While Germany was, and in a measure still is, the nursery for the world in some of the sciences and arts, other nations of Europe have used the German ideas only to reach their own maturity. They have then developed their sciences and arts in accordance with their own national tendencies and peculiarities.

The Italians have developed a distinct school of music, and also of some of the sciences—among them mathematics.

The French have a distinct school of the art of painting, and have methods and tendencies in some of the sciences which are peculiar to them and to their national characteristics.

It is true that we are a young nation, but we have already reached maturity. We ought not to follow the utilitarian ideas in education of one nation, nor the impractical ones of another. We ought to develop a system of teaching mathematics—if we find the present system wanting—which is peculiarly our own, which is in accordance with our ideals and inspirations, our temperament and environments, in short, which is American.

No matter how experienced the teacher may be, he must be sure that he has a thorough mastery of an idea before presenting it to his students, and he must plan and lay out in every detail the work which he is to take up with the class. In support of this argument, I may be permitted to state an incident of my own experience. After having taught a certain subject for a number of years, both in the classroom and in private tutoring, I felt that I was no longer in need of going over the details of the lesson before meeting the class. It happened, however, that during one year I had two sections in this subject, on the same days of the week, but at different hours. The sections did not follow each other on the different days in the same order; on some days one section and on other days the other section came first. I found that after teaching one section, I could do the work with the next section more effectively,

and go over the ground in a shorter time than with the first. To give both sections as equal advantages as possible, I made it a rule to go over the lesson before meeting the first section just as though I were a novice in teaching the subject.

The abstruse manner in which a subject is presented must not be confused with scientific treatment. The more difficult a mathematical idea, the more lucidly and clearly it ought to be presented to save the student time, strength, and often discouragement.

Mathematics is advancing at such a rapid rate that it is becoming quite impossible even for those who have high ability and much leisure to devote to its study, to obtain a knowledge of all its different branches. If this immense amount of subject matter is not presented lucidly, as is the case in some of the works of the foremost writers on mathematics, the task of acquiring mathematical knowledge is quite discouraging, even to the ablest students.

The great masters in their flights of genius perhaps ought not to be expected to descend to the level of the average mortal; but the writers of text-books, and those who record the results of their research but who do not belong to this very exclusive class, ought to present their ideas and their work in a clear and lucid manner. The late Professor Tait must have assumed that all the readers of his rather abstruse "Elementary Treatise on Quaternions," Cambridge, 1890, would bring to it the same ability, experience and knowledge in the subject which he has possessed when he wrote the book. The following quotation from this book page 110, shows his sad lack of sympathy with those who are perhaps, less gifted but who are striving for knowledge. "The words above, 'it is evident,' have been objected to by more than one correspondent. But, on full consideration, I not only leave them where they are, but put them in Italics. For they are, of course, addressed to the reader only; and it is to be presumed that, before he reaches them, he has mastered the contents of at least the previous sections which bear on this question. If, with these sections in his mind, he does not see the 'evidence,' he has begun the study of quaternions too soon."

There is among teachers a great deal of difference of opinion as to the method of conducting the work in the classroom,

especially in the advanced classes of the high school, and in the college. Some teachers favor exclusively the so-called recitation method; others believe that the best results will be obtained by explaining the subject to the class for the entire period. This method most frequently results in a formal lecture. By means of an examination in the subject at the end of the term or of the year, or by occasional tests during the term, the teacher gains information as to the standing in the subject of the members of the class. But neither of these methods of conducting the work with a class does justice to the pupil. It is the duty of the teacher to find out the state of knowledge of the student from day to day. In this way the teacher will see in what particular the student's understanding of the work is wanting. By giving the necessary explanation he will make the ideas and the work clear to the student. This method with some modifications ought to be used even in the work of the graduate and the professional school.

We can never be sure whether or not we know a thing, or understand an idea, unless we come to practice it, or to express it or to apply it. One of the means of acquiring clear ideas is to speak the ideas out or to write them out. We must have the student talk the ideas over with us. In this way only will he find out whether or not they are perfectly clear to him.

Besides getting clear ideas of the conceptions and laws of mathematics, the student must also acquire facility in performing its various operations. The acquiring of this facility is an excellent discipline, and is necessary, as stated above, if the student is to continue the study of mathematics. Part of this facility the student must acquire by means of home work. He will have to work at the examples assigned to him, until he has entire mastery of all of the operations involved and can with little effort carry them through to correct and accurate results.

The teacher must indicate the mistakes which the student has made in such work and supplement the work of the student in such a manner as to make it entirely clear to him. The teacher must continually encourage the student to ask pertinent questions and not rest content until he has mastered each point of the lesson.

Very often a student refrains from asking a question because of a somewhat indefinite idea which seems to exist in the mind

of some of them, that they have, so to speak, no claim on the services of the teacher, but that these services are rendered, in a sense, as a matter of favor. If such an opinion is prevalent, the teacher must make it clear to the student that it is his duty to give the student all the information and help he may need, and to answer such questions as will make the ideas presented entirely clear to him. If the teacher is strong in character and in mind—and only such ought to be employed—the clear understanding which the students will have of the teacher's duties and functions will not lessen their respect for him and not minimize his importance.

Since the acquisition of knowledge is, as we have stated, a matter of evolution, we must do our work from day to day. We must digest and assimilate the ideas we learn one by one, if we are to acquire clear conceptions of them, and if they are to help our mental development and increase our mental power.

Human nature is such that we are apt to do promptly what we must give an immediate account of, and postpone such work for which we are not at once accountable.

While I thoroughly believe that a person cannot do as good work at one time as he can at another, and that we must in a certain sense be disposed to do our work, yet practical life does not tolerate such tendencies and variances of the mind. In life, we have to do our day's work, whatever our personal inclinations may be. We ought therefore to train the youth to do his work from day to day, and not to give way to slight indispositions and moods, unless the mind and body are in such a state that it would be wiser to submit than to try to conquer them.

In most of the foreign universities attendance at classes is not compulsory. The student is not held accountable for his work from day to day. The work in the classroom consists of more or less formal lectures delivered by the teacher.

It is preposterous to think that a young man of eighteen or nineteen, with very little experience of the world and the things in it, and often with little maturity of mind, should consider it an affront if he be made to attend lectures and submit to being questioned, unless he does it from his own volition. To be compelled to attend lectures and recite, would, in the opinion of these students be a violation of their so much cherished "Akademische Freiheit" (academic freedom).

In some of these universities the student, whatever his age, considers himself as belonging to an exclusive and privileged class, and I cannot help but feel that he is often encouraged in this belief by the authorities of the university. If he commits an offence which would make him liable to punishment, in accordance with the law as applied to any other person, he is in some of the university towns, not judged by the properly constituted courts, but is left by them to be judged by the officers of the university. The punishment inflicted by these authorities generally consists in incarceration in the university prison (Karcer). The jolly hours spent by the student in this "Karcer" have been described in prose and verse.

The general conviction in this country that all men are equal before the law, and the consequent respect for the law, has a beneficial effect on the morals of the people.

In spite of the fact that in nearly every country in Europe, an accurate record is kept of the occupation of every inhabitant, of his previous residence, etc., I feel that the personal safety and the moral status here, where all these regulations and precautions do not exist, are much greater than in most European countries.

The system of education as it is carried on in some of the foreign universities is bound to affect the intellectual and moral status of the entire nation. If we can judge the signs of the times and predict the future, the center of gravity of intellectual powers is bound to shift from such nations to those where education is conducted in a more effective manner.

While I am not able to support my arguments by statistics, it is my own observation and experience that the number of students who leave some of the foreign universities without finishing their studies is growing larger and larger.

Such men, as a rule, become a burden to society, and are a curse to civilization. Having been accustomed to the "Akademische Freiheit," many of them are not fit to take employment and to be subservient. They augment the ranks of those who are dissatisfied with the world and the things in it. Some of them become "walking delegates," or—I rather hesitate to express it—missionaries to foreign countries.

This shall not be construed as showing on my part a lack of appreciation of the self-sacrificing men and women who are

willing to go to far-off lands and administer to those who need their help. There are men and women who have felt the calling to be missionaries, and many already while in youth have prepared themselves for their chosen life work.

The teachers in some of the foreign universities consider research in their special branch of learning, as their principal duty, and the instruction of the students a matter of secondary importance.

They pursue this course not as a result of any lack of a sense of duty, but as a matter of custom and tradition, dating from the time when only a few, and those the ablest, attended the university. These students came to the university more for the sake of getting the stimulus and the inspiration which comes from contact with great minds, than for the purpose of receiving instruction. But a great injustice is done to the student who attends a university in order to acquire knowledge and obtain information, if he does not receive instruction in his studies from those whose duty it is to give it.

To many of these investigators mankind owes a debt of gratitude for the unselfish manner in which they have devoted themselves to the advancement of science. They have by their labors, not only increased knowledge, but also have added to the comforts and conveniences of life, and often their discoveries and inventions have been the means of producing wealth. The only reward for their efforts is the pleasure and satisfaction which comes from searching for truth and finding it.

The man who makes the discovery or the invention, especially if it is along scientific lines, does not, as a rule, reap any material benefit from his abilities and labor. One of the most touching incidents I know of is that of Heinrich Hertz, the inventor of wireless telegraphy. He sacrificed his life for science, and overwork led to a premature death. Science is enriched by his labors. The comfort and convenience which his invention has added to life is hard to estimate. It is pathetic to think that his name is not even connected with the invention, and that he is not even given popular credit for his labors.

It is true that there are men who, while carrying on research in their chosen science, are still able to descend to the level of the beginner, and maintain an intellectual companionship with even the youngest student. Such teachers, I fear, are the ex-

ception. I feel that the majority of the teachers when engaged in the all-absorbing work of scientific research, are not apt to be in sympathy with the needs of the beginner in the subject.

Hegel is said to have calmly finished his "Phaenomenologie des Geistes" at Jena, on October 14, 1806, not knowing anything whatever of the battle that was raging around him. The carrying on of original investigation, especially in the pure sciences, requires the closest concentration of our mental powers which very often absorbs our physical strength and endangers our health, and may thus unfit us for the difficult task of teaching the young. I also quote in this connection from a letter of the great Jacobi:

"It is a wearisome work which I have done, wearisome work in which I am engaged. It is neither diligence nor memory which leads here to the goal, for they are here merely the subservient ministers of the moving pure thought. But tenacious, brain-splitting pondering (Nachdenken) requires more strength than the most persistent diligence. If therefore, by the constant practice of this pondering I have acquired some power in it, let it not be thought that it has been made easy for me, perhaps by a fortunate gift of nature. Wearisome, wearisome work I have had to undergo, and the anxiety of the pondering has often seriously endangered my health. The consciousness, however, of power attained, forms the best reward of the work, and is also an encouragement to continue and not to relax."

The young ought to be taught not only to husband their physical strength, but to be economical with their time as well. It is most important that the student, whether in preparing his lessons or in his reading, learn to accomplish the best results with the least expenditure of time. In other words, he must learn how to study and how to work.

The method of the spiral system which has been successfully used in the teaching of arithmetic and which ought to be applied to the teaching of each of the branches of mathematics, ought also to be followed in reading and studying any subject, especially the more profound ones. It would be a waste of time, and with some people, a source of discouragement, to attempt to read a difficult work, say on philosophy, and endeavor to have a thorough understanding of it as we go along. A good plan is to read the whole book, or a part of it carefully, without at-

tempting to thoroughly understand its most difficult passages. The second reading of the book will clarify some of these passages which are not clear at the first reading. Repeated readings will make the book clearer and clearer until a thorough understanding of the whole work is reached.

In mathematics, as in any other subject, the student must go over the lesson carefully, concentrate his mind and his attention on it, without using an undue amount of time on the more difficult parts. After going over the lesson a second time, he will find that the ideas are much clearer to him. He will have to go over the lesson again and again until he has a thorough mastery of it.

The student must learn to concentrate all of his thoughts on a piece of work in hand, and must not allow his attention to be distracted by exterior influences. He ought to be trained to think and to meditate. Meditation is a great help towards clarifying our ideas on subjects we have learned, or we have heard, or read or seen. Meditation develops and strengthens the power and the habit of thinking. In the rush of life, we do not meditate enough.

The youth must be constantly reminded and persistently urged to make the best use of his opportunities and of his time. If I am permitted to be again personal—and I feel that we certainly can speak more authoritatively about our own experience than about the experience of others—there is nothing that fills me with so much regret as to think of the days and the hours of which I have not made proper use. We feel the regret the more, as we grow older. O! for the hours wasted which never return; for the opportunities lost which never come again.

I do not, however, wish it to be understood that the young ought to be expected to employ their time and energy only in study and in such a manner as will be of immediate use and direct benefit to them.

Time spent in well directed out-door exercises, time spent in reading a good book, or in the company of a stimulating friend, in nursing good comradeship; time spent in listening to good music, in seeing a good play, or in watching manly sport, is time well spent.

It would appear that the teacher in the college or in the university is, so to speak, the court of the last resort, and is the only judge of his efforts, yet this is far from being so.

While the work of the secondary and elementary schools has been lately subject to a great deal of criticism, the work of the college has been attacked by men of affairs on whose sympathy with the college its maintenance depends, and its great mission is made possible.

These attacks have been found to be worthy of comment, even by some of the most conservative college presidents. While the work in the college depends on the preparation which the pupil receives in the elementary and secondary schools, yet the college is judged irrespective of this preparation, and only by what the finished product—the graduate of the college—is able to do and by what he represents.

Those who have been successful in life, in spite of their lack of college education, or, as in some cases without any school education because of their superior qualities of character and abilities of mind, and perhaps also because they were aided by favorable conditions, are apt to be rather skeptical of the benefits to be derived from higher education. There are some who believe that a higher education even unfits a man or woman for the work of life. I cannot help thinking that such criticisms are merely the result of experience with those college men, who have failed to make proper use of the opportunities and facilities offered by the institutions which they have attended. A person may go through college, and yet not be educated. The amount of ground covered in a subject does not determine the mental status of the learner. The inventory of the mental powers of a person cannot be measured by the number of books read, by the number of topics studied, by the number of facts learned, but only by the number of clear ideas acquired, and to what extent he is able to apply them to his own moral and physical betterment, and to the welfare of his fellowmen. Such criticisms will be silenced when the young realize the true purpose for which they go to school and college, when they avail themselves of the opportunities, and make proper use of the facilities offered for their physical, moral and intellectual development.

True education ought to teach the youth that all work is ennobling if it is done well and if it helps towards obtaining the necessary comforts and conveniences of life. That experience gives facility, and without facility the most effective results

cannot be obtained. To do our duty should be our highest ambition. The greater our opportunities, the greater our responsibilities. A man's worth ought to be measured by his character only, and the man who has been fortunate enough to enjoy the advantages of education, ought to become a missionary for all that is good and noble. He should administer his ideas by precept and example to those who have not enjoyed such advantages as he has.

Idleness and vice go hand in hand, but neither ever remains unpunished. Reward and punishment do not in general follow our actions immediately, but will surely come some day, perhaps not until the lapse of generations. Leisure is only to be enjoyed by those who have done their life's work, and who know how to use it.

True arguments must not be supported by quotations, but must be substantiated by reasoning, or by results which confirm the experience of mankind.

Education must teach the student true modesty. It must teach him, that our knowledge is little as compared with what we still have to learn; that we are but passing trifles when compared with the eternity of matter and the immensity of the universe; that unless we contribute something towards helping the world to become better than we find it, we have lived in vain.

It is the wish and the hope of parents that their children be in every way better than they are. It must be our purpose to leave the world better than we have found it. In this, and only in this way, is the progress of the world morally, intellectually and socially possible.

While a knowledge of the history of mathematics and of all other kinds of information as to the development and the philosophical aspects of the subject will broaden the teacher and make him better fit for his work, yet all this will be of little avail if he has not a thorough knowledge of the subject he is teaching, perfect facility in solving any of the examples and problems in connection with the work, and such a mastery and resourcefulness in the subject which is above the book and syllabus and as will enable him to adapt the instruction to the needs of students of different ability, and different environments.

Better text-books, a change in the number and the kind of subject taught, a better distribution of the hours for work and

the hours for recreation, both for pupils and teachers, a change in the ratio between the number of pupils and the number of teachers, will help and help a great deal if the teacher has the highest qualities of character and mind, and is able to impress his pupils with them, and use them to make the pupils follow his teachings and advice in the classroom and out. The most efficient supervision, the wisest direction by school officers, all advances in the physical equipment of the school will be of little avail, unless the government of the school is able to attract and retain men and women of the highest qualities of character and mind, well equipped for their work. The plans of the architect are of little use, unless the builder and sculptor are able to execute them and to give form to the plans.

The teacher must understand the peculiarities of the student and their difficulties, he must be able to descend to the level of the student and to enter into an intellectual companionship even with the beginner in a subject, and also with those of little ability. If the teacher will do this, then such charges as are often made by students, that while the teacher seems to have a knowledge of the subject, he does not seem able to impart it to his students, will no more be heard.

The mode of presenting a subject, the manner of handling a class cannot very well be learned from books. The description of an event is no match to actually seeing it happening. The reading of instructions is only a weak substitute for the "living word," and seeing these instructions actually carried out. No one should have the right to teach unless he attends for a certain period of time the classes in actual progress conducted by men and women who have reached a high proficiency of imparting knowledge. The candidate must have studied and be proficient in the subject of which such a class is treating, in order that he may be able to concentrate his attention and efforts on the mode of presentation, and the manner in which the class is conducted.

What we need more than anything else is to attract men and women to our profession who not only have perfect mastery of the subject they are teaching, and the ability to impart the knowledge to their pupils, but also who have the personality, the enthusiasm for their high calling, and all those qualifications of character and mind to make them an inspiration and a stimu-

lus to their students. There is no occupation in which qualities of sincerity and frankness and the most extreme straightforwardness are more important than in teaching. If the teacher is to accomplish his great task, he must have the complete confidence of his pupils. The teacher exerts the most lasting influence during the formative period of the child's character. The character of the teacher, his habits, his entire life must be models after which the student can most safely pattern his own life.

While the profession of the teacher is in a sense rather respected, yet on the whole, the teacher does not enjoy the appreciation which is accorded to members of other professions. I feel that the public at large, and especially men of affairs, are inclined to consider the teacher whose actions and activities extend only to the young, as not possessing such qualities as the men who are in daily contact with the practical world.

If the teacher is to discharge his important duties of developing the character and the mind of the young, and thus effecting the amelioration of the race, he must be well informed about the world and the things in it, about everything that is good and noble and everything that needs to be improved and remedied.

The teacher with the opportunities for self-culture and meditation, ought to be best equipped to take a leading part in every movement that will make for moral and intellectual betterment.

While the profession of the teacher has the attraction, that it gives more leisure than is enjoyed by the members of most of the other professions, it does not, on the other hand, offer such possibilities as other professions hold out to those who enter them. The teacher on entering his profession knows pretty well what limit he is able to reach; he cannot, on the whole, aspire to reach such a goal as the average American youth may set for himself when he enters other professions or business.

The tendency has prevailed at all times, and is stronger now than ever before to offer in any of the activities of life, high rewards for superior qualifications and high efficiency of service. The same tendencies ought to apply to the teaching profession also.

There is a difference in the degree of ability necessary to solve the various problems which the follower of most of the professions meets in his practice. It is not as difficult to diagnose

and cure some diseases as it to conquer others. Some engineering problems require for their solution a high degree of ability and often great ingenuity, while a moderate degree of ability is necessary to solve others. The same is true of the problems in the other professions. As far as the teaching profession is concerned, it takes as high a degree of ability, of character and mind, and of all other high qualities, to mould the character and mind of the child attending the lowest grades of the elementary schools as it does to influence the student in the highest classes of the college. In all other professions and occupations there is room for persons with moderate ability. If they perform their duties conscientiously, their services are as necessary and important as are the services of those who are engaged in the more difficult problems of their profession. In the teaching profession there is room only for those of highest ability and superior qualities of character, mind and heart, if education is to accomplish its great mission, the amelioration of the race.

Teaching is an art, and a high and noble art. It is a God-given art to cut a statue out of a piece of marble, which after all is only subject to the chisel of the sculptor. What nobler art it is to mould the character and the mind of the child —with its environments, home influence and temperament—so as to make it, morally, mentally and physically as nearly as possible, a perfect man or woman.

The public provides, on the whole, liberally for the necessities of the community. If it could be made to see the great influence and the mission of education, then the respect for education and for the teacher will rise, and he will be accorded in every way the credit which he deserves.

UNIVERSITY OF PENNSYLVANIA,
PHILADELPHIA, PA.

THE AIMS IN TEACHING ALGEBRA AND HOW TO ATTAIN THEM.*

BY C. E. BIKLÉ.

It should be the aim of algebra teaching to secure a part, and that a large one, of the values to be derived from the study of mathematics. These values are very well stated by Professor

* Abstract of paper read at meeting of Syracuse Section, December 29.

D. E. Smith and Professor J. W. A. Young, with whose books you are doubtless familiar. Granting that the securing of these values ought to be among the aims of teaching mathematics, some of the definite problems presented to the teacher of secondary mathematics are the following :

1. To interest pupils in mathematics, both as a subject by itself, and as applicable to science and to commercial and industrial life.

2. To establish valuable habits of accuracy, of verifying results, of promptness and of neatness of work, which can be carried into other lines of study and into business life.

3. To give the pupils those forms of reasoning and that training in logic that shall be helpful in other lines of work.

4. To prepare the pupils to pass with credit the examinations set by the different colleges.

5. To prepare the pupils to pursue successfully their college work in mathematics.

6. To give the pupils an interest in mathematics other than that which springs from the fact that it is necessary for their advancement in the school and their admission to college.

That one of the aims I have mentioned is not entirely reached is indicated by the following figures taken from the annual reports of the College Entrance Examination Board of the Middle States and Maryland.

	Total Number of Candidates.	Candidates from New York State.	Percentage of Candidates Receiving 60 Per Cent. or More.		
			To Quadratics.	Quadratics and Beyond.	Advanced Algebra.
1901	973	554	87	79	51
1902	1,362	818	70.4	53.2	Series 29.5 Th. equations 24
1903	1,620	892	68.8	48	Series 35 Th. equations 31.8
1904	1,817	982	59.8	59.7	Series 31.7 Th. equations 34.3
1905	2,077	1,065	43.6	31.6	Advanced Algebra 15.9
1906	2,432	1,106	59.1	53.5	Advanced Algebra 26.7
1907	3,048	1,267	48.6	34.6	Advanced Algebra 50
1908	3,250	1,235	29.6	37.7	Advanced Algebra 69.8

After making deductions for poor question-papers set by the examiners and for the fact that the board examinations are taken by many pupils who are not prepared for them, and not recommended by their teachers, I think it safe to say that there are some things to be remedied in the teaching of algebra.

College announcements frequently call attention to the inadequate preparation of students in algebra.

I cite the figures of the board because I have read many algebra papers for them in the years 1901–1908, and have a very distinct impression of the points in which pupils have failed. In factoring, for instance, improvement is noticeable in recent years, but there are still many candidates who do not seem to know what factoring means. Many leave the work unfinished, while others reach the form of an indicated product by doing great violence to the terms of the expression.

(To be continued.)

NOTES AND NEWS.

THE SECOND ANNUAL DINNER OF THE PHILADELPHIA SECTION, held on Saturday evening, February 13, proved to be a very delightful affair.

Previous to the dinner a reception was held in order that the members of the section might become better acquainted with each other.

There were fifty-nine present and Mr. Henry V. Gummere, of Drexel Institute, the chairman of the section, presided. The guest of honor was Professor David Eugene Smith, of the Teachers College, Columbia University. At the conclusion of the dinner he favored the assembly with a very interesting account of the work of the International Congress of Mathematicians, which met in Rome last summer.

Dr. Cheesman A. Herrick, principal of the new William Penn High School for Girls, made a plea for giving the pupils more practical work in mathematics.

Among other guests of the evening who gave very happy short talks were Mrs. Katherine D. Brown, of Drexel Institute, Mr. Eugene Randolph Smith, of the Polytechnic Preparatory School of Brooklyn, Dr. Fletcher Durell, of the Lawrenceville School, and Mr. George Alvin Snook, of the Central High School.

The evening was pronounced a great success, both mentally and socially.

AT THE AUTUMN MEETING OF THE PHILADELPHIA SECTION the subject for discussion was "The Original in Algebra." Papers upon the subject were presented by Dr. George B. McClellan Zerr and Mrs. Katharine D. Brown; the discussion was opened by Professor George H. Hallett.

At the spring meeting the subject to be considered is "The Original in Geometry."

MANY MEMBERS OF THE PHILADELPHIA SECTION and others well known in educational circles were pleasantly entertained, February 25, by Mr. Henry V. Gummere and his colleagues of the mathematical staff of the Drexel Institute.

MR. JOHN C. BECHTEL has been elected an instructor in mathematics, at the Central High School, Philadelphia.

THE ROCHESTER SECTION held its sixth regular meeting at the University of Rochester on Saturday, February 13, 1909. The programme consisted of the following papers:

"Report of a Discussion, in Buffalo, of the Present Regents Syllabus," by Miss Ellen D. Baker, Central High School, Buffalo.

"Tests for Convergency of Infinite Series," by C. W. Watkeys, the University of Rochester.

"Symmetry as a Basis of Construction Work in Elementary Geometry," by William Betz, East High School, Rochester.

"The Use of Squared Paper in Elementary Mathematics," by A. S. Gale, the University of Rochester.

As a result of the discussion of Miss Baker's report, the chairman was instructed to appoint a committee to draw up a list of desired changes in the present regents' syllabus, to be submitted, after adoption, to the Board of Regents; and to communicate with the New York and Syracuse sections with the idea of securing their coöperation in the movement.

Mr. Watkeys classified a number of tests for convergence, indicated how they are to be proved and considered their application to the hypergeometric series.

Mr. Betz showed that the idea of symmetry with respect to a line may be developed in a very simple manner, and that the fundamental constructions of geometry may be derived from this one idea so easily that a child in the grades could understand them.

The use of cross-section paper in the grammar schools was advocated by Professor Gale. It may well be used in drawing, in connection with the multiplication table, and especially in mensuration. Simple graphs not only interest children, but they are of such general importance as to merit consideration.

About thirty teachers attended the meeting.